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Original Article

DEVELOPMENT AND OPTIMIZATION OF EXPERIMENTAL DATA BASED MODELS FOR BAMBOO SLIVER CUTTING BY USING HUMAN POWERED FLYWHEEL MOTOR

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ABSTRACT

This paper presents the development of the experimental data based mathematical models for cutting the slivers from bamboo by means of human powered flywheel motor. There were various dependent and independent variables involved in the process of cutting the bamboo sliver by human powered flywheel motor. Therefore, apart from formulation and development of the model, the optimization was done to find the best sets of the independent variable to achieve the responses as an output. The bamboo sliver cutting machine was designed, fabricated and based on the theory of experimentation, the experiment was performed and total 108 sets of readings were recorded. In this work, the responses of three response variables such as number of slivers, processing time and resistive torque were experimentally studied and the experimental data based models for these three response variables are optimized to get the best set of independent variables involved in the bamboo sliver cutting process.

KEYWORDS: Bamboo, Sliver, HPFM, Optimization, Experimentation, Pi Terms, Dependant Variables, Independent Variables, Processing Time, Number of Slivers & Resistive Torque

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INTRODUCTION

The bamboo sliver cutting machine which is energized by human powered flywheel motor (HPFM) consists of HPFM Unit, mechanical power transmission system and bamboo sliver cutting unit (process unit). HPFM Unit is bicycle driven energy unit comprising of speed raising gear pair and a flywheel whereas mechanical power transmission system consists of a clutch and torque amplification gear pair. The schematic arrangement of the machine is shown in figure 1 whereas figure 2 (a) shows the actually designed and fabricated experimental set up of sliver cutting machine and figure 2 (b) shows the fly wheel with clutch and gearing arrangement.

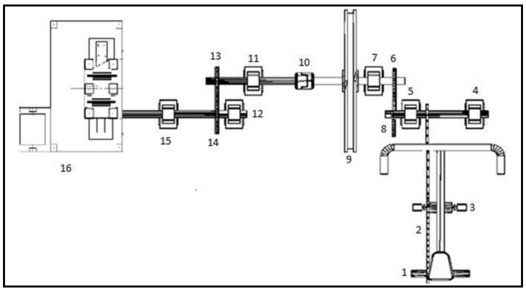

Figure 1: Schematic Arrangement of Bamboo Sliver Cutting Machine by HPFM

1 - Chain Sprocket, 2 - Chain, 3 - Pedal, 4 & 5 - Bearing for bicycle, 6 - Gear I, 7 - Bearing Flywheel

Shaft, 8 - Gear – II, 9 – Flywheel, 10 – Clutch, 11 - Bearing for flywheel shaft, 12 - Bearing for Process Unit Shaft, 13 - Gear – III, 14 - Gear – IV, 15 - Bearing for Process Unit Shaft, 16 - Process Unit.

Figure 2(a): Fabricated Experimental Set Up of Sliver Cutting Machine

Figure 2 (b): Flywheel with Clutch and Gearing Arrangement

Freely chosen pedalling rate under inertial crank load conditions at power 150 W and 250 W of a bicyclist would increase with the increase in inertial crank load [1]. There is significance of the flywheel as an energy storage device in its design and geometry. The thin rim flywheel is best to store energy and second position is taken from the flywheel rim with the web. The disc type flywheel is worst to store energy and its energy stored to flywheel-weight ratio is very low [2]. The operation of bamboo sliver cutting process involves that when an operator drives the bicycle by pedalling the mechanism while clutch is in disengage position, the flywheel is accelerated and energized which stores some energy inside it. When the pedalling is stopped, clutch is engaged and stored energy in the flywheel is transferred to the sliver cutting unit input shaft. [3-8].

The sliver cutting unit comprises of feeder, two pairs of spring loaded rollers, sliver cutter, adjusting knobs etc. [11]. When the energy from the flywheel is transferred to the sliver cutting unit by engaging the clutch, the split bamboo is fed through feeder. It enters the first pair of push-in rollers, then comes out of push-out roller pair and strikes the sliver cutter which is kept fixed and the sliver is cut. The sliver cutting immediately commences upon the clutch engagement it continues for 5 to 20 seconds until the flywheel comes to rest. There is a provision of operating the system at the speeds by properly choosing the gear ratio of a torque amplification provided on the sliver cutting unit shaft. For the experimentation purpose, the gear ratios of 1/2, 1/3 and 1/4 are taken in to practice.

METHODOLOGY OF THEORY OF EXPERIMENTATION

The method of Design of Experiment (DoE) is applied in this work. Design of Experiment (DoE) has its advantages such as it can increase the result achieved, reduce variability, and make the result closer to the target, shorten the amount of time and reduce the cost. DoE can be used for finding optimal parameter of any processes [9], like bamboo sliver cutting by means of HPFM. The approach of Design of Experimentation [10] is applied for formulating generalized experimental data based model which includes identification of variables, reduction of independent variables by dimensional analysis, test planning (comprising of determination of test envelope, test points, test sequence and experimental plan), physical design of experimental set up, execution of experimentation, purification of experimental data, model formulation, reliability of models, model optimization. The various dependent and independent variables involved in the bamboo sliver cutting operation by HPFM are shown in table 1.

Table 1: Various Dependent and Independent Variables

S. N	Variables	Unit	MLT	Dependent/ Independent
1	T_r = Resistive Torque	N-mm	ML^2T^{-2}	Dependant
2	t_p = Processing Time	Second	T	Dependant
3	n = No. of Slivers	--	$M^0L^0T^0$	Dependant
4	E_f= Flywheel Energy	N-mm	ML^2T^{-2}	Independent
5	ω_f = Angular speed of flywheel	Rad/s	T^{-1}	Independent
6	t_f = Time required to speed up the flywheel	Second	T	Independent
7	G = Gear Ratio	--	$M^0L^0T^0$	Independent
8	g = Acceleration due to Gravity	mm/s^2	LT^{-2}	Independent
9	L_b= Length of Bamboo split	mm	L	Independent
10	W_b= Width of Bamboo split	mm	L	Independent
11	t_b = Thickness of Bamboo split	mm	L	Independent
12	C_H = Horizontal Center Distance between Roller Pairs	mm	L	Independent
13	C_V = Vertical center distance between roller pairs	mm	L	Independent
14	L_{rc}= Distance between Roller Center to Cutter Tip	mm	L	Independent
15	E_b = Modulus of Elasticity of Bamboo	N/mm^2	$ML^{-1}T^{-2}$	Independent
16	E_c = Modulus of Elasticity of Cutter	N/mm^2	$ML^{-1}T^{-2}$	Independent
17	Φ_c = Cutting Angle of Cutter	Degree	-	Independent

The Buckingham's pi theorem was used for dimensional analysis and the various pi (π) terms were formed for dependent and independent variables involved in the sliver cutting process. Further, to reduce the complexity and to obtain the simplicity in the behavior of the sliver cutting phenomenon, the pi terms of the independent variables were reduced by reduction of variables method as per theories of experimentation [10]. Various pi terms in reduced form for independent variables and pi terms for dependent variables are given in the table 2. The test points and test envelope were also decided in the design of experimentation.

Table 2: Pi Terms in Reduced Form for Independent Variables

Variables	Pi Terms	Pi terms Equations	Description
Independent Variables	π_1	$\pi_1 = \frac{E_f}{L_b^3 E_b}$	The term related to the energy of the flywheel
	π_2	$\pi_2 = \omega_f \sqrt{\frac{L_b}{g}}$	The term related to the angular speed of the flywheel
	π_3	$\pi_3 = t_f \sqrt{\frac{g}{L_b}}$	The term related to time required to speed up the flywheel
	π_4	$\pi_4 = G$	The term related to gear ratio
	π_5	$\pi_5 = \left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)$	The term related to Machine's geometrical parameters
	π_6	$\pi_6 = \frac{E_c}{E_b}$	The term related to elasticity of materials
	π_7	$\pi_7 = \varphi_c$	The term related to cutting angle of cutter
Dependent / Response Variables	π_{D1}	$\pi_{D1} = t_P \sqrt{\frac{g}{L_b}}$	The term related to processing time
	π_{D2}	$\pi_{D2} = n$	The term related to number of slivers
	π_{D3}	$\pi_{D3} = \frac{T_r}{L_b^3 E_b}$	The term related to resistive torque

EXPERIMENTATION

The classical plan of experimentation [10]is used to carry out the experimentation. In the classical plan of experimentation all the independent pi terms except one are maintained constant at their planned fixed level values and the said independent pi terms under consideration is to be varied over its widest possible range as decided by test envelope. The experimentation is carried out to cover the entire test envelope and all the test points within the test envelope.

During experimentation, the bamboo split of three varying lengths i.e. 1.5 feet, 2.0 feet and 2.5 feet and different diameter ranges i.e. 30 mm to 40 mm, 40 mm to 50 mm and 50 mm to 60 mm having different widths and thickness are processed in the machine at four different speeds i.e. 300 rpm, 400 rpm, 500 rpm and 600 rpm and at three different gear ratios 1/2, 1/3, and 1/4. Thus the different varieties of bamboo are used during experimentation for monitoring the actual feasibility of the machine. During experimentation processing time, resistive torque, no. of slivers, time of flywheel to speed up etc. are measured using specially designed electronic kit viz. instrumentation system as shown in figures 3(a) and 3(b).

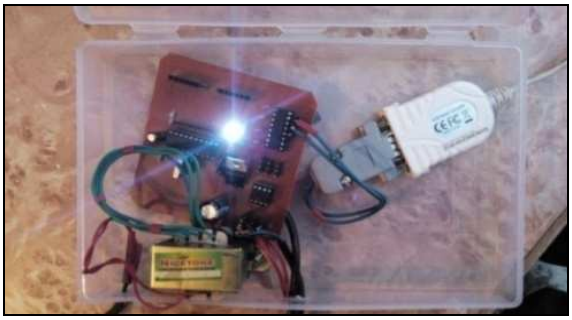

Figure 3(a): Designated Instrumentation System

Figure 3(b): Experimental Reading Generation

The total 108 sets of the readings were taken. The experimental plan and sample observations of the experiment are given in following table 3. Table 4 gives the experimental sample observations for independent parameters related to bamboo sliver cutting unit and table 5 shows experimental sample observations for energy unit and response variables of slivering process.

Table 3: Sample Experimental Plan and observations for Bamboo Sliver Cutting Operation by HPFM (when Gear Ratio = 0.5)

S. N	Dimeter Range of Bamboo (D_b) mm	Gear Ratio (G)	Length of Bamboo Split (L_b)ft	Length of Bamboo Split (L_b)mm	Speed (N) rpm	Width of Bamboo Split (W_b) mm	Thickness of Bamboo Split (t_b) mm	Time of Flywheel to Speed up (ω_f) sec	Processing Time (t_p) sec	No. of slivers (n)
1	30-40	0.5	1.5	457.5	300	28.7	5.3	31.4	40	3
2	30-40	0.5	1.5	457.5	400	29.3	5.3	41.87	50	4
3	30-40	0.5	1.5	457.5	500	28.9	5.2	52.33	50	5
4	30-40	0.5	1.5	457.5	600	29.1	5.4	62.8	65	6
5	30-40	0.5	2	610	300	24.1	6.9	31.4	40	2

Table 4: Sample Observation Table for Slivering Unit

S. N	Independent Parameters Related to Bamboo Sliver Cutting Unit														
	D_b mm	W_b mm	t_b mm	L_b ft	L_b mm	N rpm	ω_f rad/s	G	E_b N/mm^2	E_c N/mm^2	C_H mm	C_V mm	L_{rc} mm	φ_c deg	φ_c rad
1	30-40	28.7	5.3	1.5	457.5	300	31.4	0.5	20000	206000	115	65	45	15	0.2616
2	30-40	29.3	5.3	1.5	457.5	400	41.87	0.5	20000	206000	115	65	45	15	0.2616
3	30-40	28.9	5.2	1.5	457.5	500	52.33	0.5	20000	206000	115	65	45	15	0.2616
4	30-40	29.1	5.4	1.5	457.5	600	62.8	0.5	20000	206000	115	65	45	15	0.2616
5	30-40	24.1	6.9	2	610	300	31.4	0.5	20000	206000	115	65	45	15	0.2616

Table 5: Sample Observation Table for Energy Unit and Response Variables

S. N	Independent Parameters Related to Energy Unit				Dependent Parameters		
	I_f	g	E_f	t_f	t_p	Avg. Resistive torque T_r	n
	kg.m^2	mm/s^2	N-m	sec	sec	N-mm	
1	3.44	9810	1695.851	30	40	21500	3
2	3.44	9810	3014.847	30	50	20840	4
3	3.44	9810	4710.698	35	50	23580	5
4	3.44	9810	6783.405	60	65	24107.69	5
5	3.44	9810	1695.851	30	40	24475	2

DEVELOPMENT OF THE MODELS

During experimentation, the data generated belongs to the dependent variables. The data of various independent variables are gathered during experimentation. In case of bamboo sliver cutting energized by HPFM, there were seven independent variables in reduced form and three dependent variables. All these variables are represented by pi terms corresponding to each variable. The mathematical model is nothing but formulating correlation between these independent pi terms and a dependent pi term. The mathematical model is called as generalized experimental data based model as it is formulated on the data generated through experimentation. Equations 1, 2, 3 and 4 are the developed mathematical models for processing time, number of slivers, average resistive torque and total resistive torque respectively.

$$t_P = 5.15\,X10^{-10}\sqrt{\frac{L_b}{g}}\left\{\begin{matrix} \left(\frac{E_f}{L_b^3 E_b}\right)^{0.2889}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{0.1564}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.1769} \\ (G)^{-0.3499}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0371}\left(\frac{E_c}{E_b}\right)^{-3.1676}(\varphi_c)^{-30.5644} \end{matrix}\right\} \quad (1)$$

$$n = 174.1406\left\{\begin{matrix} \left(\frac{E_f}{L_b^3 E_b}\right)^{0.5082}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{-0.3148}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.1208} \\ (G)^{-0.5089}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{-0.1823}\left(\frac{E_c}{E_b}\right)^{1.3087}(\varphi_c)^{-1.4871} \end{matrix}\right\} \quad (2)$$

$$T_{ravg.} = 1.08E + 10(L_b^3 E_b)\left\{\begin{matrix} \left(\frac{E_f}{L_b^3 E_b}\right)^{0.7824}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{-1.0005}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.351} \\ (G)^{-1.4423}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0889}\left(\frac{E_c}{E_b}\right)^{4.289}(\varphi_c)^{23.515} \end{matrix}\right\} \quad (3)$$

$$T_{rtotal} = 7.68\,X10^{10}(L_b^3 E_b)\left\{\begin{matrix} \left(\frac{E_f}{L_b^3 E_b}\right)^{0.7188}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{-1.2769}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{0.0101} \\ (G)^{-1.3708}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0265}\left(\frac{E_c}{E_b}\right)^{-7.5718}(\varphi_c)^{6.7265} \end{matrix}\right\} \quad (4)$$

In order to interpret the mathematical models formed earlier, the models are analyzed quantitatively using appropriate techniques. The behavior of the models is evaluated through graphical presentation in order to justify how the real phenomena work on account of appropriate interaction of independent π-terms [12, 13]. In this model, there are seven independent π-terms and three dependent π-terms. The 3D graph is plotted by taking dependent π-terms on Z-axis, whereas from seven independent π-terms, there are combined and a product obtained is taken on X-axis. The remaining four independent π-terms are combined by taking a product and taken on Y-axis. This graphical analysis is shown in figure 4 to figure 7 for all response variables involved in human powered bamboo slivering process.

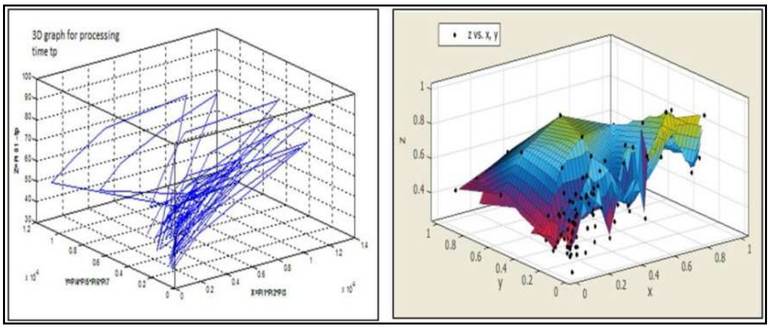

Figure 4: Graphical Analysis of Processing Time vs Pi Terms

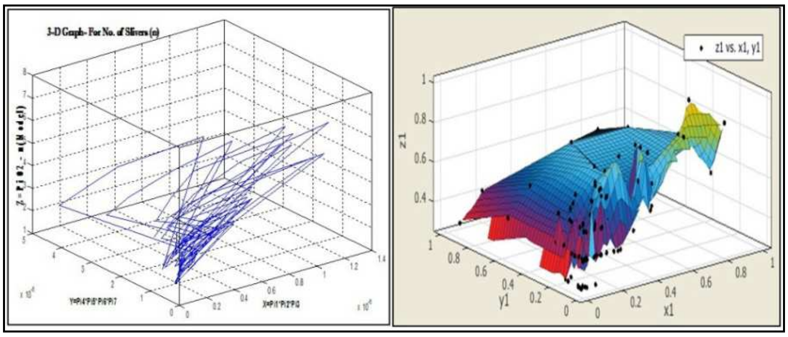

Figure 5: Graphical Analysis of Number of Slivers vs Pi Terms

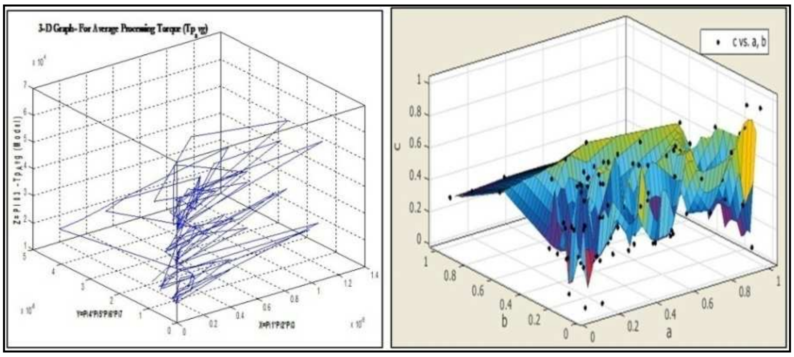

Figure 6: Graphical Analysis Average Resistive Torque vs Pi Terms

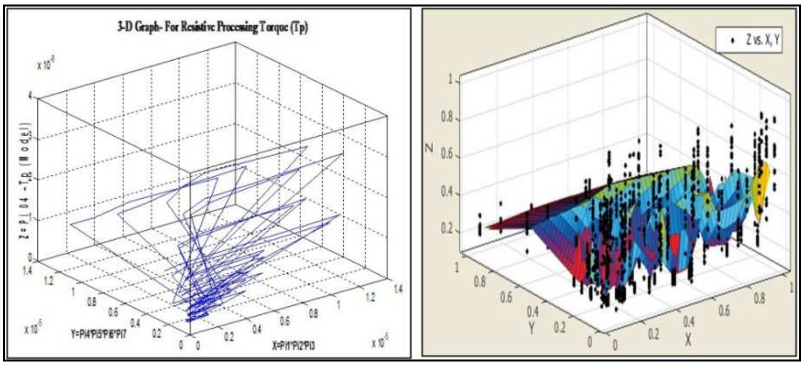

Figure 7: Graphical Analysis Total Resistive Torque vs Pi Terms

From Figure 4, it is observed that there are 11 peaks in graph of Z i.e. processing time vs. X. There must be in all 22 mechanisms which are responsible for giving these 11 peaks. Figure 5 shows that there are 11 peaks in graph of Z i.e. number of slivers vs. X. There must be in all 22 mechanisms which are responsible for giving these 11 peaks. Figure 6

gives the 14 peaks in graph of Z i.e. resistive torque-average vs. X, hence there must be in all 28 mechanisms are responsible for giving these 14 peaks and similarly from Figure 7, it has been observed that there are 16 peaks in graph of Z i.e. resistive torque-total vs. X, therefore there must be in all 32 mechanisms are responsible for giving these 16 peaks. This is based on reasoning while deciding number of physical mechanisms prevalent in any complex phenomenon [14].

OPTIMIZATION OF THE MODELS

Optimization is the act of obtaining the best result under given circumstances. The ultimate goal of decisions like designing, constructing and maintaining the engineering system is either to minimize the effort require or to maximize the desired benefit. Since the effort required or the benefit desired in any practical situation can be expressed as function of certain decision variables, optimization is process of finding the conditions that give the maximum or minimum value of function [15]. The ultimate objective of optimizing the mathematical models for bamboo sliver cutting process was to find an alternative with the most cost effective or highest achievable performance under the constraints, by maximizing desired response factors like number of bamboo slivers and minimizing the factors like processing time of cutting the slivers and resistive torque with respect to independent variables in the slivering process. While adopting optimization technique, analysis and design are merely taken care wherein analysis is done for determining the response of the HPFM driven bamboo sliver cutting machine to the certain combination of input or independent parameters and design part is also taken care of to define the system of HPFM driven bamboo sliver cutting machine. Optimization made convenient to find the minimum or maximum of the objective function, while satisfying all the required design constraints [16].

The mathematical models were developed for the sliver cutting phenomenon. The ultimate objective of this work was not merely developing the models, but to find out the best set of independent variables, which will result in maximization/minimization of the objective functions. In this case there are three different models corresponding to the Processing time (t_p), Number of slivers (n), and Resistive torque (T_r). There are thus three objective functions corresponding to these models. The objective functions for processing time and resistive torque required for sliver cutting from bamboo need to be minimized and the objective functions for number of slivers need to be maximized. The models have non-linear form; hence it is to be converted into a linear form for optimization purpose. This can be achieved by taking the log of both the sides of the model. The linear programming technique is applied which is detailed as below.

$$\pi_{D1} = K_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1} \times (\pi_4)^{d1} \times (\pi_5)^{e1} \times (\pi_6)^{f1} \times (\pi_7)^{g1} \quad (5)$$

Taking log of both the sides of the equation, we have,

$$log\, \pi_{D1} = logK_1 + a1\log(\pi_1) + b1\log(\pi_2) + c1\log(\pi_3) + d1\log(\pi_4) + e1\log(\pi_5) + f1\log(\pi_6) + g1\log(\pi_7) \quad (6)$$

Let, $log\, \pi_{D1}$= Z, $logK_1$= k_1, $\log(\pi_1) = X_1$, $\log(\pi_2) = X_2$, $\log(\pi_3) = X_3$, $\log(\pi_4) = X_4$, $\log(\pi_5) = X_5$, $\log(\pi_6) = X_6$, $\log(\pi_7) = X_7$.

Then the linear model in the form of first degree polynomial can be written as under:

$$Z = k + (a_1 \times X_1) + (b_1 \times X_2) + (c_1 \times X_3) + (d_1 \times X_4) + (e_1 \times X_5) + (f_1 \times X_6) + (g_1 \times X_7) \quad (7)$$

More specifically, an optimization consists of a function, termed the objective function which are response variables like number of slivers, processing time to cut the slivers and resistive torque that describes the goal of the bamboo sliver cutting process which needs to be minimized or maximized; a set of input / independent variables termed the

design variables, whose optimum combination is required and a set of constraints that may be related to the configuration and physical characteristics [17]. Design variable analysis is the process of finding the minimum or maximum of some parameter which may be called the objective function. For the design to be acceptable, it must also satisfy a certain set of specified requirements called constraints. The need is to find the constrained minimum or maximum of the objective function [16].

Thus, Equation (7) constitutes for the optimization or to be very specific maximization for the purpose of formulation of the problem. The constraints can be the boundaries defined for the various independent pi terms involved in the function. During the experimentation the ranges for each independent pi terms have been defined, so that there will be two constraints for each independent variable. The maximum and minimum values of a dependent pi term $\Pi_{D1}$are denoted by Π_{D1max} and Π_{D1min} respectively then the first two constraints for the problem were obtained by taking log of these quantities and by substituting the values of multipliers of all other variables except the one under consideration equal to zero. The log of the limits are defined, as C_1 and C_2 i.e. $C_1=\log(\Pi_{D1max})$ and $C_2=\log(\Pi_{D1min})$. Thus, the Equations of the constraints are as under:

$$1 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \leq C1$$

$$1 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \geq C2 \tag{8}$$

The other constraints are likewise found as:

$$0 \times X_1 + 1 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \leq C3$$

$$0 \times X_1 + 1 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \geq C4$$

$$0 \times X_1 + 0 \times X_2 + 1 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \leq C5$$

$$0 \times X_1 + 0 \times X_2 + 1 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \geq C6$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 1 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \leq C7$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 1 \times X_4 + 0 \times X_5 + 0 \times X_6 + 0 \times X_7 \geq C8$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 1 \times X_5 + 0 \times X_6 + 0 \times X_7 \leq C9$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 1 \times X_5 + 0 \times X_6 + 0 \times X_7 \geq C10$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 1 \times X_6 + 0 \times X_7 \leq C11$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 1 \times X_6 + 0 \times X_7 \geq C12$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 1 \times X_7 \leq C13$$

$$0 \times X_1 + 0 \times X_2 + 0 \times X_3 + 0 \times X_4 + 0 \times X_5 + 0 \times X_6 + 1 \times X_7 \geq C14 \tag{9}$$

This linear programming problem is solved to get the minimum value of Z. The values of the independent pi terms are then obtained by finding the antilog of the values of Z, X_1, X_2, X_3, $X_{4,}$ $X_{5,}$ X_6 and X_7. The actual values of the multipliers and the variables were found. This is solved as a linear programming problem using the MS Solver available in MS Excel. The actual problem in this case is stated as below:

Thus the actual problem is to minimize Z (i.e. minimum processing time (tp), where

$$t_P = 5.15\ X10^{-10}\sqrt{\frac{L_b}{g}}\left\{\begin{matrix}\left(\frac{E_f}{L_b^3 E_b}\right)^{0.2889}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{0.1564}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.1769}\\ (G)^{-0.3499}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0371}\left(\frac{E_c}{E_b}\right)^{-3.1676}(\varphi_c)^{-30.5644}\end{matrix}\right\}$$

Taking log of both the sides of the equation,

$\mathrm{Log}(\Pi_{D1}) = \log(5.15\ X10^{-10}) + \log\left(\sqrt{\frac{L_b}{g}}\right) + 0.2889 \times \log\left(\frac{E_f}{L_b^3 E_b}\right) + 0.1564 \times \log\left(\omega_f\sqrt{\frac{L_b}{g}}\right) - 0.1769 \times \log\left(t_f\sqrt{\frac{g}{L_b}}\right) - 0.3499 \times \log(G) + 0.0371 \times \log\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right) - 3.1676 \times \log\left(\frac{E_c}{E_b}\right) - 30.5644 \times \log(\Phi c)$.

$$Z = K + K_1 + a \times X_1 + b \times X_2 + c \times X_3 + d \times X_4 + e \times X_5 + f \times X_6 + g \times X_7 \quad (10)$$

and

$Z = \log(5.15\ X10^{-10}) + \log(0.248037) + 0.2889\ \log(\pi_1) + 0.1564.\log(\pi_2) - 0.1769.\log(\pi_3) - 0.3499\ \log(\pi_4) + 0.0371.\log(\pi_5) - 3.1676.\ \log(\pi_6) - 30.5644.\log(\pi_7)$

$Z = -9.28781 + -0.60548 + 0.2889 \times X_1 + 0.1564 \times X_2 - 0.1769 \times X_3 - 0.3499 \times X_4 + 0.0371 \times X_5 - 3.1676 \times X_6 - 30.5644 \times X_7$

Z (Processing Time:Π_{D1} min) = $-9.28781 + -0.60548 + 0.2889\ x\ X_1 + 0.1564\ x\ X_2 - 0.1769.\ x\ X_3 - 0.3499\ x\ X_4 - 0.0371\ x\ X_5\ 3.1676\ x\ X_6 - 30.5644\ x\ X_7$ (11)

Similarly for Number of Slivers, Where

$$n = 174.1406\left\{\begin{matrix}\left(\frac{E_f}{L_b^3 E_b}\right)^{0.5082}\left(\omega_f\sqrt{\frac{L_b}{g}}\right)^{-0.3148}\left(t_f\sqrt{\frac{g}{L_b}}\right)^{-0.1208}\\ (G)^{-0.5089}\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{-0.1823}\left(\frac{E_c}{E_b}\right)^{1.3087}(\varphi_c)^{-1.4871}\end{matrix}\right\}$$

Taking log of both the sides of the equation,

$\mathrm{Log}(\Pi_{D2}) = \log(174.1406) + 0.5082\ \log\left(\frac{E_f}{L_b^3 E_b}\right) - 0.3148\ \log\left(\omega_f\sqrt{\frac{L_b}{g}}\right) - 0.1208\ \log\left(t_f\sqrt{\frac{g}{L_b}}\right) - 5089\ \log(G) - 0.1823\ \log\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right) + 1.3087\log\left(\frac{E_c}{E_b}\right) - 1.4871\ \log(\Phi c)$

$Z = K + K_1 + a\ x\ X_1 + b\ x\ X_2 + c\ x\ X_3 + d\ x\ X_4 + e\ x\ X_5 + f\ x\ X_6 + g\ x\ X_7$

and

$Z = \log(174.1406) + 0.5082\ \log(\pi_1) - 0.3148\ \log(\pi_2) - 0.1208\ \log(\pi_3) - 5089\ \log(\pi_4) - 0.1823\ \log(\pi_5) + 1.3087\log(\pi_6) - 1.4871\ \log(\pi_7)$

$$Z = 2.2409 + 0.5082\ X_1 - 0.3148\ X_2 - 0.1208\ X_3 - 0.5089\ X_4 - 0.1823\ X_5 + 1.3087\ X_6 - 1.4871\ X_7 \quad (12)$$

Also for Resistive Torque-Average, Where

$$T_r = 1.08E10(L_b^3 E_b)\left\{\begin{matrix} \left(\frac{E_f}{L_b^3 E_b}\right)^{0.7824} \left(\omega_f \sqrt{\frac{L_b}{g}}\right)^{-1.0005} \left(t_f \sqrt{\frac{g}{L_b}}\right)^{-0.351} \\ (G)^{-1.4423} \left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0889} \left(\frac{E_c}{E_b}\right)^{4.289} (\varphi_c)^{23.515} \end{matrix}\right\}$$

Taking log of both the sides of the equation,

$$Log(\Pi_{D3}) = \log(1.08E+10) + \log(L_b^3 E_b) + 0.7824\log\left(\frac{E_f}{L_b^3 E_b}\right) - 1.0005\log\left(\omega_f\sqrt{\frac{L_b}{g}}\right) - 0.351\log\left(t_f\sqrt{\frac{g}{L_b}}\right) - 1.4423 \log(G) + 0.0889 \log\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right) + 4.289 \log\left(\frac{E_c}{E_b}\right) + 23.515 \log(\Phi c)$$

$Z = K + K_1 + a \times X_1 + b \times X_2 + c \times X_3 + d \times X_4 + e \times X_5 + f \times X_6 + g \times X_7$, and

$Z = \log(1.08E+10) + \log (5.11E+12) + 0.7824\log(\pi_1) - 1.0005 \log(\pi_2) - 0.351\log(\pi_3) - 1.4423 \log(\pi_4) + 0.0889 \log(\pi_5) + 4.289 \log(\pi_6) + 23.515 \log(\pi_7)$

$$Z=10.0317+12.70817) + 0.7824 X_1 - 1.0005 X_2 - 0.351 X_3 - 1.4423 X_4 + 0.0889 X_5 + 4.289 X_6 + 23.515 X_7 \quad (13)$$

And for Resistive Torque-Total

$$T_r = 7.68 X10^{10}(L_b^3 E_b)\left\{\begin{matrix} \left(\frac{E_f}{L_b^3 E_b}\right)^{0.7188} \left(\omega_f \sqrt{\frac{L_b}{g}}\right)^{-1.2769} \left(t_f \sqrt{\frac{g}{L_b}}\right)^{0.0101} \\ (G)^{-1.3708} \left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right)^{0.0265} \left(\frac{E_c}{E_b}\right)^{-7.5718} (\varphi_c)^{6.7265} \end{matrix}\right\}$$

Taking log of both the sides of the Equation, we get

$$Log(\Pi_{D3}) = \log(7.68 X10^{10}) + \log(L_b^3 E_b) + 0.7188 \log\left(\frac{E_f}{L_b^3 E_b}\right) - 1.2769 \log\left(\omega_f\sqrt{\frac{L_b}{g}}\right) + 0.0101 \log\left(t_f\sqrt{\frac{g}{L_b}}\right) - 1.3708 \log(G) + 0.0265 \log\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right) - 7.5718 \log\left(\frac{E_c}{E_b}\right) + 6.7265 \log(\Phi c)$$

$Z = K + K_1 + a \times X_1 + b \times X_2 + c \times X_3 + d \times X_4 + e \times X_5 + f \times X_6 + g \times X_7$, and

$$Z = \log(7.68 X10^{10}) + \log(L_b^3 E_b) + 0.7188 \log\left(\frac{E_f}{L_b^3 E_b}\right) - 1.2769 \log\left(\omega_f\sqrt{\frac{L_b}{g}}\right) + 0.0101 \log\left(t_f\sqrt{\frac{g}{L_b}}\right) - 1.3708 \log(G) + 0.0265 \log\left(\frac{W_b t_b C_H C_V L_{rc}}{L_b^5}\right) - 7.5718 \log\left(\frac{E_c}{E_b}\right) + 6.7265 \log(\Phi c)$$

$Z = \log(7.68 X10^{10}) + \log (5.11E+12) + 0.7188 \log(\pi_1) - 1.2769 \log(\pi_2) + 0.0101 \log(\pi_3) - 1.3708 \log(\pi_4) + 0.0265 \log(\pi_5) - 7.5718 \log(\pi_6) + 6.7265 \log(\pi_7)$

$$Z=10.8855+12.70817) + 0.7188 X_1 - 1.2769 X_2 + 0.0101 X_3 - 1.3708 X_4 + 0.0265 X_5 - 7.5718 X_6 + 6.7265 X_7 \quad (14)$$

Thus from Eq. (11), (12), (13) and (14),

Z (Processing Time:Π_{D1} min) = -9.28781+ -0.60548+ 0.2889 x X_1 +0.1564. x X_2 -0.1769. x X_3 -0.3499 x X_4 +0.0371. x X_5 3.1676. x X_6 -30.5644. x X_7

Z (No. of slivers: Π_{D2} max)=2.2409+0.5082 X_1 - 0.3148 X_2-0.1208 X_3 -0.5089 X_4 – 0.1823 X_5+1.3087 X_6- 1.4871 X_7

Z (Resistive torque-avg: Π_{D3} max)=10.0317+12.70817)+ 0.7824 X_1 -1.0005 X_2 - 0.351 X_3 - 1.4423 X_4 + 0.0889 X_5 + 4.289 X_6 + 23.515 X_7

Z (Resistive torque-total: Π_{D3} max)=10.8855+12.70817)+ 0.7188 X_1- 1.2769 X_2+0.0101 X_3 -1.3708 X_4 +0.0265 X_5- 7.5718 X_6 +6.7265 X_7

Subjected to the following constraints

$1\times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \leq -8.45076$

$1 \times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \geq -9.71836$

$0 \times X1+ 1 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \leq 1.243245$

$0 \times X1+ 1 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \geq 0.831291$

$0 \times X1+ 0 \times X2 + 1 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \leq 2.44379$

$0 \times X1+ 0 \times X2 + 1 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \geq 1.855745$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 1 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \leq -0.30103$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 1 \times X4 + 0 \times X5 + 0 \times X6 + 0 \times X7 \geq -0.60206$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 1 \times X5 + 0 \times X6 + 0 \times X7 \leq -5.05772$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 1 \times X5 + 0 \times X6 + 0 \times X7 \geq -6.49524$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 1 \times X6 + 0 \times X7 \leq 1.012837$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 1 \times X6 + 0 \times X7 \geq 1.01283$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 1 \times X7 \leq -0.58225$

$0 \times X1+ 0 \times X2 + 0 \times X3 + 0 \times X4 + 0 \times X5 + 0 \times X6 + 1 \times X7 \geq -0.58225$ (15)

The above problem is solved by using MS solver to get the values of X_1, X_2, X_3, X_4, X_5, X_6, X_7 and Z. Thus Π_{D1min} = Antilog of Z and corresponding to this value of the Π_{D1min} the values of the independent pi terms were obtained by taking the antilog of X_1, X_2, X_3, X_4, X_5, X_6, X_7 and Z. A similar procedure is adopted to optimize the models for Π_{D2max} and Π_{D3min} [18-20]. And the optimized values of response variables $\Pi_{D1min,}$ Π_{D2max} and Π_{D3min} are tabulated in the tables 6(a) and 6(b).

Table 6(a): Optimized Values of Model for Processing Time and Number of Slivers

	Optimized Values of Model for Processing Time (π_{D1min})		Optimized Values of Model for Number of Slivers (π_{D2max})	
	Log values of π terms	Antilog of π terms	Log values of π terms	Antilog of π terms
Z	1.449045934	28.12198252	1.1421987	13.87390448
X_1	-9.718361797	1.91266E-10	-8.450755556	3.54197E-09
X_2	0.831290694	6.780952366	0.831290694	6.780952366
X_3	2.443790205	277.8370793	1.855744575	71.7372254
X_4	-0.301029996	0.5	-0.602059991	0.25
X_5	-6.495239563	3.19713E-07	-6.495239563	3.19713E-07
X_6	1.012837225	10.3	1.012837225	10.3
X_7	-0.582251598	0.261666667	-0.582251598	0.261666667

Table 6(b): Optimized Values of Model for Average and Total Resistive Torque

	Optimized Values of Model for Average Resistive Torque (π_{D3min})		Optimized Values of Model for Total Resistive Torque (π_{D3min})	
	Log Values of π Terms	Antilog of π Terms	Log Values of π Terms	Antilog of π Terms
Z	3.543750001	3497.437812	-9.013803264	9.69E-10
X_1	-9.718361797	1.91266E-10	-9.718361797	1.91266E-10
X_2	1.243245064	17.50834372	1.243245064	17.50834372
X_3	2.443790205	277.8370793	1.855744575	71.7372254
X_4	-0.301029996	0.5	-0.301029996	0.5
X_5	-6.495239563	3.19713E-07	-6.495239563	3.19713E-07
X_6	1.012837225	10.3	1.012837225	10.3
X_7	-0.582251598	0.261666667	-0.582251598	0.261666667

CONCLUSIONS

The machining properties (viz. processing time, resistive torque) of bamboo sliver cutting phenomenon by means of HPFM are established through theory of experimentation. The data in the present work are collected by performing actual experimentation due to which the finding of the present study truly represents the degree of interaction of various independent variables in the human powered bamboo sliver cutting phenomenon. The trends for the behavior of the models are demonstrated by graphical analysis and it has been noted that the mathematical models can be successfully used for the computation of dependent terms for a given set of independent terms in case of the sliver cutting process. The models were formulated for both average resistive torque of total 108 sets of observations and total resistive torques of 1380 observations of all 108 sets for bamboo sliver cutting operation by HPFM and are optimized.

It is noted that the phenomenon is complex, because of variation in the dependent pi terms which are in a fluctuating form mainly due to continuous variation in the angular speed of the process unit shaft. The variation in the process unit shaft is exponentially dropping. This in turn is due to linearly varying load torque on the process unit shaft due to the process resistance and inertia resistances which are likely to be instantaneous speed depend upon the variation in human energy, non-linear cross section of bamboo and quality of bamboo in 0.5 to 5 seconds.

From analysis of indices of the models, it is observed that for the model of processing time (π_{D1}),the absolute index of π_1 is highest i.e. 0.2889. The factor π_1 is related to the energy of the flywheel which is the most influencing term in this model. The value of this index is positive, indicating involvement of energy of the flywheel has strong impact on π_{D1} and π_{D1} is directly varying with respect to π_1 i.e. energy of the flywheel. Model of number of slivers (π_{D2}) indicates that the absolute index of π_6 is highest (1.3087), which is related to the ratio of elasticity of material and it is the most influencing

term in this model. This index is a positive showing ratio of elasticity of the material has strong impact on π_{D2} and π_{D2} is directly varying with respect to elasticity of the material. From the model of average resistive torque, it is analyzed that the π_7 has highest absolute index viz. 23.515 and it is related to cutting angle of cutter which is the most influencing term in this model. Involvement of cutting angle of cutter has strong impact on π_{D3} and π_{D3} is directly varying with respect to cutting angle of cutter. Total Resistive torque model shows that the absolute index of π_7 is highest viz.6.7265 and hence it also has same impact as in case of model of average resistive torque.

The optimized values of dependent pi terms of bamboo sliver cutting operation are π_{D1} =28.12 seconds, π_{D2} = 13.87 and π_{D3} =3497.43 N-mm (for average resistive torque),π_{D3} =4736.210661 N-mm (for total resistive torque). For these optimized values of all response variables, the values of all pi terms π_1 to π_7 and the values of input parameters for bamboo sliver cutting operation energized by HPFM are found and given in table 7:

Table 7: Optimized Values of all pi Terms and Input Parameters for Bamboo Sliver Cutting Operation

Optimized Value of Response Variables	π_1	π_2	π_3	π_5	Input parameters for bamboo sliver cutting operation						
					E_f (N-mm)	N (rpm)	t_f (sec)	G (π_4)	t_b (mm)	E_c/E_b (π_6)	$\varphi_c$rad (π_7)
Processing time, t_p = 28.12 seconds	1.91E-10	6.78	277.83	3.19E-07	1695.85	300	60	0.5	8.1	10.3	0.26
Number of slivers, n = 13.87	3.54E-09	6.78	71.73	3.19E-07	6783.40	300	20	0.25	8.1	10.3	0.26
Average resistive torque, $T_{r\text{-}avg}$ = 3497.43 N-mm	1.91E-10	17.50	277.83	3.19E-07	1695.85	600	60	0.5	8.1	10.3	0.26
Total resistive torque, $T_{r\text{-}total}$ = 4736.21 N-mm	1.91E-10	17.50	71.73	3.19E-07	1695.85	600	20	0.5	8.1	10.3	0.26

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